

Prior knowledge of mathematics: Short answers

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Short answers to the test problems

Terms

1.
 - a) 14
 - b) $\frac{1}{9}$
 - c) -16
 - d) 4

2.
 - a) $3x + 13y + 3z$
 - b) $18m + 6q$
 - c) $10a - b$
 - d) $60a^5b^3$
 - e) $x^4 - y^4$

3.
 - a) $p^2 + 2pq + q^2$
 - b) $4x^2 + 12xy + 9y^2$
 - c) $x^2 - 2xy + y^2$
 - d) $4a^2 - 12a^2x + 9a^2x^2$
 - e) $a^2 - 4$
 - f) $25x^2y^2 - 9x^2z^2$

4.
 - a) $5a^2(1 - 2a - 5a^2)$
 - b) $3a(x - a)(x + 3a)$

5.
 - a) $\frac{7}{9b}$
 - b) $\frac{1}{abc}$
 - c) $\frac{2b}{a - b}$
 - d) $\frac{p}{p - 1}$
 - e) - 1

6. a) $\frac{20b^2y}{10a^2b^2x}$

b) $\frac{25ab^2}{10a^2b^2x}$

7. a) $\frac{3x}{5}$

b) $\frac{5x-8y}{a}$

c) $\frac{5x}{6}$

d) $\frac{a^2-c}{ab}$

e) $\frac{a^2+ab-b}{a^2-b^2}$

f) $\frac{t+6}{3t}$

8. a) $\frac{5}{2}$

b) $\frac{1}{6ab}$

c) $-\frac{2d}{3}$

d) $6q$

e) $4b+3q$

f) $6a^2b^2c$

g) $x+2$

h) $\frac{x}{c}$

i) $\frac{1}{ab}$

j) xy

k) r

9. a) $a^{24}b$

b) $a^{-10}b^{10}$

c) $a^{-4}b^{9/5}$

10. a) The expression is defined for all real numbers x .

b) $x = -2$

c) $x < -3$

d) $-2 \leq x \leq 2$

Sets

11. a) $B \cup C = \{2, 3, 4, 5, 6, 8\}$
 b) $B \cap C = \{2, 4\}$
 c) $(A \cup B) \setminus C = \{1, 6, 7, 8\}$
12. a) true
 b) false
 c) true
 d) true
 e) false

Equations

13. a) $x = \frac{11}{2}$
 b) $x = 9$
 c) The equation has no solution.
 d) The solutions of the equation depend on the values of the parameters a and c:
 if $c \neq -1$:
 The equation has exactly one solution, which is $x = \frac{c-2a}{c+1}$
 if $c = -1$ and $a = -1/2$:
 The equation has infinitely many solutions. Each $x \in \mathbb{R}$ is a solution.
 if $c = -1$ and $a \neq -1/2$:
 The equation has no solution.
 e) $x_1 = -6, x_2 = 2$
14. a) $x > \frac{13}{2}$
 b) The inequality has infinitely many solutions. They consist of all x which are in the ranges $-1 < x < 2$ and $x > 7/2$.
15. a) $x_1 = 0, x_2 = -1$
 b) $x_1 = 5, x_2 = -1$
 c) $x_1 = 3, x_2 = \frac{1}{2}$
 d) $x = 3$
 e) The equation has no solution.
 f) $x = -3$
16. a) $x_{1k} = \frac{\pi}{6} + k \cdot 2\pi$ and $x_{2k} = \frac{5\pi}{6} + k \cdot 2\pi$ ($k \in \mathbb{Z}$)
 b) $x_{1k} = \frac{5\pi}{12} + k \cdot \pi$ and $x_{2k} = \frac{3\pi}{4} + k \cdot \pi$ ($k \in \mathbb{Z}$)

c) $x_{1k} = k \cdot \pi$ and $x_{2k} = \frac{\pi}{2} + k \cdot 2\pi$ ($k \in \mathbb{Z}$)

17. a) $x = 3$

b) $x = 4$

c) $x = -4$

d) $x = \frac{\ln(4)}{3}$

e) $x = 0$

18. a) $(x, y) = (5, -2)$

b) $(x, y)_1 = (-11, 7)$

$(x, y)_2 = (1, 1)$

Geometry/Trigonometry

19. a) i) $b = \sqrt{c^2 - a^2}$

ii) $b = 12$

b) i) $b = \sqrt{a^2 + c^2 - 2c\sqrt{a^2 - h_c^2}}$

ii) $b = \sqrt{17}$

20. $b = \frac{2}{\sqrt{3}}, c = \frac{4}{\sqrt{3}}, \alpha = 60^\circ, \beta = 30^\circ$

Functions

21. a) $f(0) = -4$
 $f(-4) = -16$

b) $x = \frac{4}{3}$

22. a) $f(-1) = -2$

b) $f(2) \approx 2.8$

c) $x_1 = -3$
 $x_2 = 1$

d) $x_1 \approx -2.5$
 $x_2 \approx 0.3$

23. a) $y = f(x) = -\frac{9}{4}x + \frac{1}{2}$

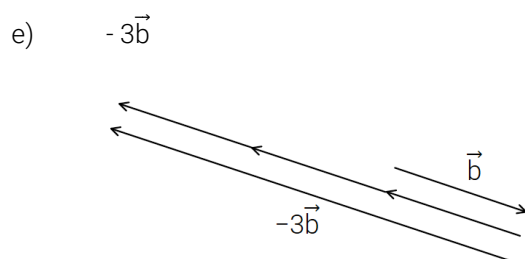
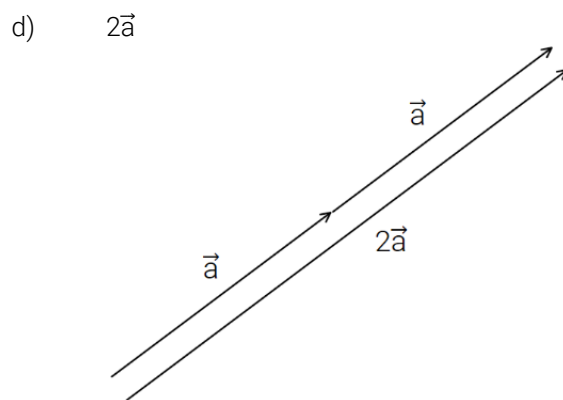
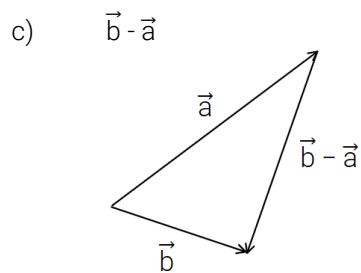
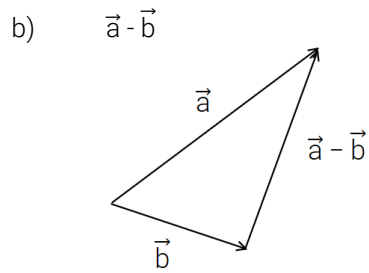
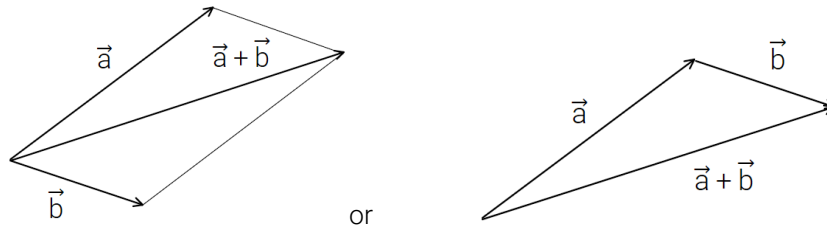
b) $S_y \left(0 \left| \frac{1}{2} \right. \right)$

c) $S_x \left(\frac{2}{9} \left| 0 \right. \right)$

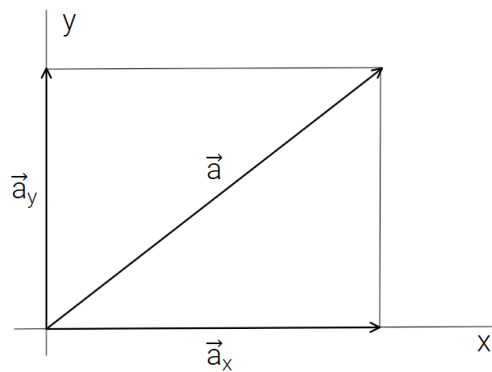
24. a) $y = f(x) = -x^2 + 8x - 8$
b) $y = f(x) = (x - 1)^2 - 8$
25. P(3|4)
26. a) true
b) true
c) true
d) false
e) true
f) true
g) false
h) false
i) true
j) true
27. a) $g(x) = 2 \sin(4x - 8)$
b) $g(x) = 2 \sin(4x + 8)$
c) $g(x) = 2 \sin(4x) + 2$
d) $g(x) = 2 \sin(4x) - 2$
e) $g(x) = 2 \sin(2x)$
f) $g(x) = 2 \sin(8x)$
g) $g(x) = 4 \sin(4x)$
h) $g(x) = \sin(4x)$
28. a) by 3 units in the positive x-direction
b) by 2 units in the negative x-direction
29. a) $f(f(x)) = x^4$
b) $g(g(x)) = 4x - 3$
c) $f(g(x)) = 4x^2 - 4x + 1$
d) $g(f(x)) = 2x^2 - 1$

Vectors

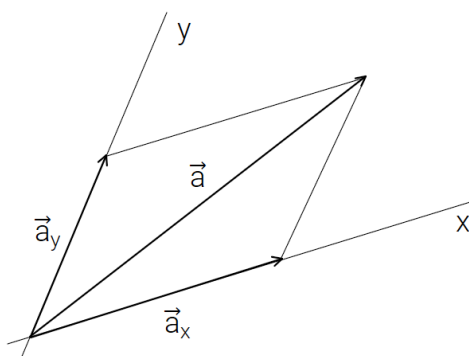
30. a) $\vec{a} + \vec{b}$



31. a)



b)



32. a) $\vec{a} + \vec{b} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

b) $\vec{a} - \vec{b} = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$

c) $2\vec{a} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$

d) $-3\vec{b} = \begin{pmatrix} 0 \\ -3 \\ 9 \end{pmatrix}$

e) $-3\vec{a} + 4\vec{b} = \begin{pmatrix} -3 \\ 10 \\ -18 \end{pmatrix}$

f) $|\vec{a}| = 3$

g) $|\vec{a} - 2\vec{b}| = 9$

33. $r = 3, s = -2, \vec{c} = 3 \cdot \vec{a} - 2 \cdot \vec{b}$